

# Polarization of the Recoil Proton for Proton Compton Scattering Below the Pion Production Threshold

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*Dedicated to Prof. Dr. K. MOLIÈRE on his 60-th birthday*

The polarization of the recoil proton for Compton scattering on protons below the pion production threshold, which arises from fourth order Feynman diagrams, is calculated by using the second order Compton amplitude in the unitarity condition. It is assumed that the second order amplitude can be approximated by the one proton intermediate state contribution. The polarization does not exceed 0.1% in the entire range of the scattering angle.

## I. Introduction

Shortly after the discovery of the violation of CP invariance in the  $2\pi$  decay of the  $K_2^0$  meson<sup>1</sup>, BERNSTEIN, FEINBERG, and LEE<sup>2</sup> suggested that this might be due to a violation of  $T$  invariance by the electromagnetic interaction of strongly interacting particles (hadrons). One of the processes, in which such a violation of time reversal invariance might show up, is the scattering of photons on protons. If  $T$  invariance is valid, then, to lowest order in the electromagnetic interaction, the polarization of the recoil protons should vanish for photon energies below the pion production threshold.

In a model calculation, LIPSHUTZ<sup>3</sup> showed that a polarization of several percent could result from violation of  $T$  invariance in the proton Compton scattering<sup>4</sup>.

On the other hand, even if  $T$  invariance is valid, there should be a polarization of the recoil protons which is proportional to the electromagnetic fine-structure constant,  $\alpha = (137.036)^{-1}$ . This polarization could be of the order of percent. It is the aim of this paper to compute the polarization due to radiative corrections to proton Compton scattering, assuming  $T$  invariance to be valid. We get the non hermitian part of the transition matrix from the unitarity condition by approximating the lowest order Compton amplitude by the one proton intermediate state contribution. We find an absolute value of polarization perpendicular to the scattering plane which is less than 0.1% for all scattering angles. This small absolute value is due to the fact that the polarization is proportional to  $(\omega/M)^2$ , where  $\omega$  is the photon energy,  $M$  the proton mass,

and  $\omega/M \leq 0.14$  below the pion production threshold.

In Sect. II we give the constraints on the transition matrix necessary to guarantee space- and time-inversion invariance. In Sect. III the unitarity condition is stated. In Sect. IV we give the lowest order amplitudes and the results.

## II. Space- and Time Inversion Invariance, and Hermiticity

We define the transition matrix  $T_{\gamma p \rightarrow \gamma p}$  as in Ref. <sup>6</sup>.

$$S = 1 - i(2\pi)^4 \delta^4(k + p - k' - p') \cdot \sqrt{\frac{M^2}{4E\omega E'\omega'}} T_{\gamma p \rightarrow \gamma p} \quad (1)$$

where  $k, p, \omega, E$  ( $k', p', \omega', E'$ ) are the four-momenta and energies of the initial (final) photon and proton, respectively.  $M$  is the proton mass. Then, in the center of momentum (c.m.) system the differential cross section for the Compton process is:

$$\frac{d\sigma}{d\Omega} = \left| \frac{1}{4\pi} \frac{M}{E + \omega} T_{\gamma p \rightarrow \gamma p} \right|^2. \quad (2)$$

By using a definite representation of Dirac spinors and  $\gamma$ -matrices one can write:

$$T_{\gamma p \rightarrow \gamma p} = T(\mathbf{p}', \mathbf{k}', \boldsymbol{\epsilon}', s'; \mathbf{p}, \mathbf{k}, \boldsymbol{\epsilon}, s) = \chi_{s'}^\dagger \mathcal{M}(\mathbf{p}', \mathbf{k}', \boldsymbol{\epsilon}'; \mathbf{p}, \mathbf{k}, \boldsymbol{\epsilon}) \chi_s \quad (3)$$

where  $\boldsymbol{\epsilon}, s$  ( $\boldsymbol{\epsilon}', s'$ ) are the photon polarization vector and the proton spin in the initial (final) state, respectively, and  $\chi$  is a two-component spinor.

$$\mathcal{M}(\mathbf{x}'; \mathbf{x}) = F(\mathbf{x}'; \mathbf{x}) 1 + i \boldsymbol{\sigma} \cdot \mathbf{G}(\mathbf{x}'; \mathbf{x}) \quad (4)$$



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The components of  $\sigma$  are the Pauli matrices;  $F$  is a complex scalar function;  $\mathbf{G}$  is a complex vector valued function; and,  $\mathbf{x}$  and  $\mathbf{x}'$  denote the set of vectors  $\{\mathbf{p}, \mathbf{k}, \boldsymbol{\epsilon}\}$  and  $\{\mathbf{p}', \mathbf{k}', \boldsymbol{\epsilon}'\}$ , respectively. The polarization of the recoil proton along the direction defined by the unit vector  $\hat{n}$  is given by:

$$P_{\hat{n}} = \frac{\text{Tr } \mathfrak{M}^\dagger \sigma \cdot \hat{n} \mathfrak{M}}{\text{Tr } \mathfrak{M}^\dagger \mathfrak{M}}, \quad (5)$$

$$P_{\hat{n}} = i \frac{F^* \mathbf{G} \cdot \hat{n} - F \mathbf{G}^* \cdot \hat{n} + \mathbf{G} \times \mathbf{G}^* \cdot \hat{n}}{F F^* + \mathbf{G} \cdot \mathbf{G}^*} \quad (6)$$

where  $z^*$  denotes the complex conjugate of  $z$  and  $\mathbf{a} \times \mathbf{b}$  the vector product of the two vectors  $\mathbf{a}$ ,  $\mathbf{b}$ . Tr means the trace. It follows immediately that  $P_{\hat{n}} = 0$  for  $F^* = F$ ,  $\mathbf{G}^* = \mathbf{G}$ .

Invariance against space- and time-inversion place restrictions on the functions  $F$  and  $\mathbf{G}$ :

Space inversion invariance:

$$\begin{aligned} F(\mathbf{x}'; \mathbf{x}) &= F(-\mathbf{x}'; -\mathbf{x}), \\ \mathbf{G}(\mathbf{x}'; \mathbf{x}) &= \mathbf{G}(-\mathbf{x}'; -\mathbf{x}). \end{aligned} \quad (7)$$

Time reversal invariance:

$$\begin{aligned} F(\mathbf{x}'; \mathbf{x}) &= F(-\mathbf{x}; -\mathbf{x}'), \\ \mathbf{G}(\mathbf{x}'; \mathbf{x}) &= -\mathbf{G}(-\mathbf{x}; -\mathbf{x}'). \end{aligned} \quad (8)$$

If, in addition, the  $T$ -matrix would be hermitian:

$$\begin{aligned} F(\mathbf{x}'; \mathbf{x}) &= F^*(\mathbf{x}; \mathbf{x}'), \\ \mathbf{G}(\mathbf{x}'; \mathbf{x}) &= -\mathbf{G}^*(\mathbf{x}; \mathbf{x}'). \end{aligned} \quad (9)$$

Only these three conditions together imply reality of  $F$  and  $\mathbf{G}$ .

### III. Unitarity

Under the invariance requirements (7) and (8) the expression (6) for the polarization is different from zero only if the  $T$ -matrix has a non vanishing non-hermitian part, and this part is given by the unitarity condition:

$$-i(T_{if}^* - T_{fi}) = (2\pi)^4 \sum_j \delta^4(p + k - Q_j) \frac{T_{jf}^* T_{ji}}{Q_j(E_j)} \quad (10)$$

where i, f, j, stand for initial, final, intermediate state,  $Q_j$  is the total four momentum of state j, and  $Q_j(E_j)$  is some kinematic factor, which enters the definition of the matrix elements  $T_{ji}$ ,  $T_{jf}$ . The sum is over all states j which are connected to the initial and final states and which are allowed by four momentum conservation. The last condition forbids any hadronic intermediate state as long as the en-

ergy of the incoming photon is less than the threshold energy for single pion production (which is about 150 MeV in the laboratory frame). For such energies we see from Eq. (10) that

$$\frac{(T - T^\dagger)_{fi}}{(T + T^\dagger)_{fi}} \sim \alpha$$

by expanding both sides in powers of  $\alpha$ . Moreover, the non-hermitian part is given by the *lowest order* Compton amplitude. So, to get the recoil proton polarization to order  $\alpha$ , we have only to insert the lowest order Compton amplitude into the sum of the unitarity Eq. (10) and we need not compute higher order diagrams<sup>7</sup>.

### IV. Choice of Lowest Order Amplitudes and Results

Let us write  $F = F_1 + iF_2$ ;  $\mathbf{G} = \mathbf{G}_1 + i\mathbf{G}_2$ , and

$$\begin{aligned} F_j &= \alpha F_j^{(1)} + \alpha^2 F_j^{(2)} + \dots, \\ \mathbf{G}_j &= \alpha \mathbf{G}_j^{(1)} + \alpha^2 \mathbf{G}_j^{(2)} + \dots, \end{aligned} \quad j = 1, 2$$

Then from (10) it follows that  $F_2^{(1)} = 0$ ;  $\mathbf{G}_2^{(1)} = 0$ , and, using (7) and (8),

$$\begin{aligned} F_2^{(2)}(\mathbf{x}'; \mathbf{x}) &= -\frac{1}{(4\pi)^2} \frac{M\omega}{E+\omega} \sum_{\hat{k}''} \int d\Omega_{\hat{k}''} \\ &\cdot [F^{(1)}(\mathbf{x}''; \mathbf{x}') F^{(1)}(\mathbf{x}''; \mathbf{x}) \\ &+ \mathbf{G}^{(1)}(\mathbf{x}''; \mathbf{x}') \cdot \mathbf{G}^{(1)}(\mathbf{x}''; \mathbf{x})], \end{aligned} \quad (11)$$

$$\begin{aligned} \mathbf{G}_2^{(2)}(\mathbf{x}'; \mathbf{x}) &= -\frac{1}{(4\pi)^2} \frac{M\omega}{E+\omega} \sum_{\hat{k}''} \int d\Omega_{\hat{k}''} \\ &\cdot [\mathbf{G}^{(1)}(\mathbf{x}''; \mathbf{x}) F^{(1)}(\mathbf{x}''; \mathbf{x}') - \mathbf{G}^{(1)}(\mathbf{x}''; \mathbf{x}') F^{(1)}(\mathbf{x}''; \mathbf{x}) \\ &+ \mathbf{G}^{(1)}(\mathbf{x}''; \mathbf{x}') \times \mathbf{G}^{(1)}(\mathbf{x}''; \mathbf{x})]. \end{aligned} \quad (12)$$

Here the sum is over the two intermediate photon polarization vectors and the integration is over the unit vector  $\hat{k}''$ . Integrating over the  $\delta$ -function in Eq. (10), we explicitly made use of the relation  $\mathbf{p} = -\mathbf{k}$ . Therefore all quantities are now in the c.m. system. Furthermore,  $\omega = \omega' = \omega''$  in the c.m. system. For energies below the pion production threshold, the main contribution to the functions  $F^{(1)}$ ,  $\mathbf{G}^{(1)}$  comes from the one proton intermediate state (Fig. 1). If expanded in powers of  $\omega/M$ , up to the linear terms the contribution of these diagrams coincides with the expression of the low energy theorem<sup>9, 10</sup>. If we are satisfied to compute the polarization up to terms of relative order  $\omega/M$ , which means up to about 15% for  $\omega \sim m_\pi$ <sup>11</sup>, we can use for  $F^{(1)}$  and  $\mathbf{G}^{(1)}$  the amplitudes given by the

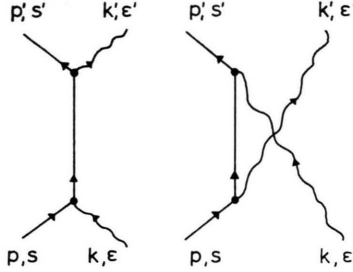


Fig. 1. Diagrams for the one proton intermediate state.

low energy theorem. In the c.m. system they are <sup>12</sup>:

$$F^{(1)}(\hat{k}', \hat{\epsilon}'; \hat{k}, \hat{\epsilon}; \omega) = -\frac{4\pi}{M} \cdot [(1 - (\omega/M) \cos \Theta) \hat{\epsilon}'(\hat{k}') \cdot \hat{\epsilon}(\hat{k}) + (\omega/M) (\hat{k}' \times \hat{\epsilon}') \cdot (\hat{k} \times \hat{\epsilon})], \quad (13)$$

$$G^{(1)}(\hat{k}', \hat{\epsilon}'; \hat{k}, \hat{\epsilon}; \omega) = -\frac{4\pi\omega}{M^2} \left[ \left( \frac{1}{2} - \frac{2\mu M}{e} \right) \hat{\epsilon} \times \hat{\epsilon}' + \frac{1}{2} \left( \frac{2\mu M}{e} \right)^2 (\hat{k}' \times \hat{\epsilon}') \times (\hat{k} \times \hat{\epsilon}) - \frac{\mu M}{e} (\hat{k}' \cdot \hat{\epsilon} \hat{k} \times \hat{\epsilon}' - \hat{k} \cdot \hat{\epsilon} \hat{k}' \times \hat{\epsilon}) \right] \quad (14)$$

where  $\hat{k}' = \omega^{-1} \mathbf{k}'$ ,  $\hat{k} = \omega^{-1} \mathbf{k}$ ,  $\hat{\epsilon}' = \boldsymbol{\epsilon}'$ , and  $\hat{\epsilon} = \boldsymbol{\epsilon}$  are all unit vectors and  $\hat{\epsilon}' \cdot \hat{k}' = \hat{\epsilon} \cdot \hat{k} = 0$ .  $e$  is the proton charge and  $\mu$  the total magnetic moment:

$$2\mu M/e = 2.792782 \pm 0.000017^{13}.$$

We have neglected the pion exchange amplitudes (Fig. 2). Here, the expansion parameter is  $\omega/m_\pi$  rather than  $\omega/M$ , and for  $\omega \sim m_\pi$  we cannot use the low energy theorem. It is not difficult to compute the contribution  $P_n^{(\pi^0)}$  to the polarization coming

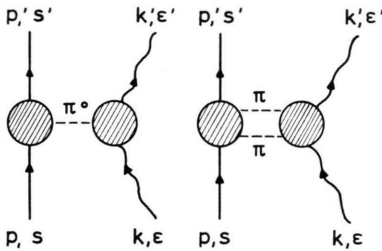


Fig. 2. The one and two pion exchange diagrams.

from the  $\pi^0$ -exchange. The result is:

$$|P_n^{(\pi^0)}| < 3 \times 10^{-6} \text{ for } \hat{n} = \hat{k} \times \hat{k}' / \sin \Theta,$$

and for all scattering angles and  $0 \leq \omega \leq m_\pi$ . So we certainly can neglect the  $\pi^0$ -exchange amplitude, and very likely all other pion exchange amplitudes too.

Now one sees that  $P \sim a(\omega/M)^2$ , i. e. a value between  $10^{-4}$  and  $10^{-3}$ , which certainly is too small to be measured. Thus we conclude that if a polarization of the recoil proton of a few percent is measured in proton Compton scattering below the pion production threshold, this will not be due to radiative corrections. However, since it is now easy enough to compute the polarization as a function of the scattering angle we may as well do it.

Inserting (13) and (14) into the right-hand sides of (11) and (12) one finds:

$$F_2^{(2)}(\hat{k}', \hat{\epsilon}'; \hat{k}, \hat{\epsilon}; \omega) = -\frac{8\pi}{3} \frac{1}{E+\omega} \frac{\omega}{M} \cdot \hat{\epsilon} \cdot \hat{\epsilon}' + 0((\omega/M)^2), \quad (15)$$

$$G_2^{(2)}(\hat{k}', \hat{\epsilon}'; \hat{k}, \hat{\epsilon}; \omega) = -\frac{8\pi}{3} \frac{1}{E+\omega} \left( \frac{\omega}{M} \right)^2 \cdot \left[ \left( 1 - 5 \frac{\mu M}{e} \right) \hat{\epsilon} \times \hat{\epsilon}' + \frac{\mu M}{e} (\hat{\epsilon}' \cdot \hat{k} \hat{k} \times \hat{\epsilon} - \hat{\epsilon} \cdot \hat{k}' \hat{k}' \times \hat{\epsilon}') \right] + 0 \left( \left( \frac{\omega}{M} \right)^3 \right).$$

For  $\hat{n} = (\hat{k} \times \hat{k}') / \sin \Theta$ , where  $\Theta$  is the scattering angle in the c.m. system, one gets from (6), (15), and (16)

$$P_n(\omega, \Theta) = \frac{2}{3} \alpha \frac{M}{E+\omega} \left( \frac{\omega}{M} \right)^2 \sin \Theta \frac{\left( 3 \frac{2\mu M}{e} - 1 \right) \cos \Theta - \left( \frac{2\mu M}{e} \right)^2 + 0 \left( \frac{\omega}{M} \right)}{1 + \cos^2 \Theta + 2 \frac{\omega}{M} \cos \Theta \sin^2 \Theta + 0 \left( \left( \frac{\omega}{M} \right)^2 \right)} \quad (17)$$

after averaging over the initial and summing over the final photon polarization vectors.

We expect (17) to be correct within about 15%. For  $\omega = 130$  MeV the maximum of  $|P_n(\omega, \Theta)|$  occurs at  $\Theta \sim 108^\circ$  and is equal to  $7.6 \cdot 10^{-4}$ .

- <sup>1</sup> J. H. CHRISTENSON, J. W. CRONIN, V. L. FITCH, and R. TURLAY, Phys. Rev. Letters **13**, 138 [1964].
- <sup>2</sup> J. BERNSTEIN, G. FEINBERG, and T. D. LEE, Phys. Rev. **139**, B 1650 [1965].
- <sup>3</sup> NELSON R. LIPSHUTZ, Phys. Rev. **158**, 1491 [1967].
- <sup>4</sup> A measurement of the polarization was planned at STANFORD <sup>5</sup>.
- <sup>5</sup> B. WIK, private communication.
- <sup>6</sup> A. P. CONTOGOURIS, Nuovo Cim. **25**, 104 [1962].
- <sup>7</sup> This method of computation has been used by BARUT and FRONSDAL <sup>8</sup> to get the spin-orbit correlations in muon-electron and electron-electron scattering.

- <sup>8</sup> A. O. BARUT and C. FRONSDAL, Phys. Rev. **120**, 1871 [1960].
- <sup>9</sup> F. E. LOW, Phys. Rev. **96**, 1428 [1954].
- <sup>10</sup> M. GELL-MANN and M. L. GOLDBERGER, Phys. Rev. **96**, 1433 [1954].
- <sup>11</sup> We use units  $\hbar=c=1$ .
- <sup>12</sup> See Eq. (III. 3) in Ref. <sup>6</sup>; in the rest frame of the nucleon, the terms linear in  $\omega$  are absent in Equation (13).
- <sup>13</sup> Particle Data Group, A. RITTENBERG et al., Rev. Mod. Phys. **43**, No 2, Part II, S 15 [1971].